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Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2019

Fourth Semester

Core Course—VECTOR, CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

[Common for B.Sc. Mathematics Model I, II and B.Sc. Computer Applications]

(2013—2016 Admissions)

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. Find the unit tangent vector to the curve $r(t) = 6t^3\, i - 2t^3 j - 3t^3 k,\ 1 \le t \le 2$.
2. Give an example of a conservative field.
3. Find the divergence of $F = \left(x^2 - y\right) i + \left(xy - y^2\right) j$.
4. Find the gradient field of $f(x, y, z) = xy + yz + xz$.
5. State Gauss divergence theorem.
6. Find the plane through $(0, 0, 1), (2, 0, 0)$ and $(0, 3, 0)$.
7. Give an example of an equation for which $\alpha = 2$ and $\beta = 3$ are 3 -multiple roots.
8. Write the formula for method of False position.
9. Find two numbers a and b such that a real root of $x^3 - x - 4 = 0$ lies between a and b.
10. In Newton-Raphson method to find a real root of an equation $f(x) = 0$, what we are replacing the part of the curve between $\left(x_0, f(x_0)\right)$ and the X-axis, where x_0 is the initial approximation to a root ?

$(10 \times 1 = 10)$

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Find the directional derivative of $f(x, y, z) = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction of $2i - 3j + 6k$.

12. Find the unit normal vector to the curve $r(t) = 4\sin t\, i + 4\cos t\, j + 3t\, k$.

13. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, prove that $\nabla \times (r^n \vec{r}) = \vec{0}$.

14. Show that curl (grad ϕ) = $\vec{0}$ where ϕ is a scalar point function of x, y, z.

15. Evaluate $\int_C xy\, dy - y^2\, dx$, where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

16. Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $(1, 2, 4)$.

17. Form an equation whose roots are the negatives of the roots of $x^3 - 6x^2 + 8x - 9 = 0$.

18. Solve the equation $x^3 + 6x + 20 = 0$, one root being $1 + 3i$.

19. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking the second term.

20. Solve the equation $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

21. Obtain a root correct to 3 decimal places using bisection method for $x^3 - x - 4 = 0$.

22. Using Newton-Raphson method, obtain the iterative formula to find 1/N.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \nabla\left(x^3 + y^3 + z^3 - 3xyz\right)$.

24. Find the value of the constant 'a' such that the vector field

$\vec{F} = \left(axy - z^3\right)i + (a-2)x^2\, j + (1-a)\, xz^2\, k$ is irrotational.

25. Find the flux of $F = (x - y)\, i + xj$ across the circle $x^2 + y^2 = 1$ in the xy-plane.

26. Using the line integral, compute the work done by the force $F = (2y + 3)\, I + xz\, J + (yz - x)\, K$ when is moves a particle from the point $(0, 0, 0)$ to the point $(2, 1, 1)$ along the curve $x = 2t^2, y = t, z = t^3$.

27. If α, β, γ be the roots of the equation $x^3 - px^2 + qx - r = 0$, form the equation whose roots are

$\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}, \alpha\beta + \frac{1}{\gamma}$.

28. Find the equation whose roots are 3 times the roots of $6x^3 + 2x^2 - 4x + 1 = 0$.

29. Solve the equation $x^3 - 7x^2 + 36 = 0$, given that one root is double of another.

30. Find the root of the equation $2x = \cos x + 3$ correct to 3 decimal places by iteration method.

31. Using Newton-Raphson method, evaluate $(30)^{-\frac{1}{5}}$ correct to four decimal places.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the constant a, b and c such that $\bar{F} = (x + y + az)\,i + (bx + 2y - z)\,j + (x + cy + 2z)\,k$ is irrotational. Also find a scalar function ϕ such that $\bar{F} = \nabla\phi$.

(b) If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, prove that $\nabla \frac{1}{r} = -\frac{1}{r^3}\vec{r}$.

33. (a) Use Stoke's theorem to evaluate $\int_C F.\,dr$ if $F = xzi + xyj + 3xzk$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, traversed counterclockwise.

(b) Find, by Green's theorem, the value of $\int_C \left(x^2\,y\,dx + ydy\right)$ where **C** is the closed curve formed by $y^2 = x$ and $y = x$ between $(0, 0)$ and $(1, 1)$.

34. (a) Solve $x^4 + 4x^3 - 5x^2 - 8x + 6 = 0$ given that sum of two roots is zero.

(b) Solve by Ferrari's method : $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$.

35. (a) Use Newton-Raphson method to obtain a root correct to 3 decimal places for the equation $4(x - \sin x) = 1$.

(b) Find the root of the equation $xe^x = \cos x$ using the regula-falsi method correct to four decimal places.

$(2 \times 15 = 30)$